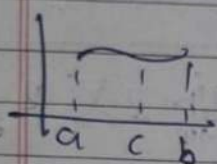


Simple Integration

$$\textcircled{1} \int_a^b f(x) dx = \int_a^b f(t) dt$$

$$\textcircled{2} \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\textcircled{3} \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$



King's rule

$$\textcircled{4} \int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$x = (a+b-x)$

Where to apply King's rule

$$I = \int_a^b \frac{f(x) dx}{f(x) + f(a+b-x)}$$

→ here answer will be $\Rightarrow \frac{b-a}{2}$

$$I = \int_a^b x \phi(x) dx$$

odd function $= f(-x) = -f(x)$

even function $= f(-x) = f(x)$

common example of odd function

$$1. f(x) = \ln\left(\frac{a-x}{a+x}\right)$$

$$\int_a^a f(x) dx = \begin{cases} 0 & \text{if it is an odd function} \\ 2 \int_{-a}^a f(x) dx & \text{if even function} \end{cases}$$

$$\int_{-a}^a \frac{dx}{1+k^{g(x)}} \quad g(x) \rightarrow \text{odd}$$

then $I = a$

$$\int_{-a}^a \frac{f(x) dx}{1+k^{g(x)}} \quad \begin{matrix} f(x) \rightarrow \text{even} \\ g(x) \rightarrow \text{odd} \end{matrix}$$

then $I = \int_{-a}^a f(x) dx$

$$\int_0^{\pi/2} \sin^2 x dx = \int_0^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$$

Integration of Modulus

* solve by breaking modulus or solve by making graph and finding area

* if there is one normal and other in modulus then solve by putting $\pm$ ve & $-$ ve then break and then integrate

Integration by Periodic

$$\int_0^{nT} f(x) dx = n \int_0^T f(x) dx \quad \text{function}$$

$$\int_a^{a+T} f(x) dx = \int_0^T f(x) dx$$

$$\int_m^{nT} f(x) dx = (n-m) \int_0^T f(x) dx$$

$$\int_0^{\pi} |\sin x| dx = \int_0^{\pi} |\cos x| dx = 2$$

Integration of Greatest Integer function

$$\int_0^n [x] dx = \frac{(n-1)n}{2}$$

$$\int_0^n \{x\} dx = \frac{n}{2}$$

$$\int_0^n [x] dx = (n-1)$$

$$\int_0^n \{x\} dx = \frac{n}{2}$$

if limits are decimal

take the thing in
of G.I.F as $f(x)$

now put in value of the
limit in $f(x)$ the write all
the ~~numbers~~ integers between
 $f(\text{lowest limit})$ & $f(\text{highest limit})$
equal to $f(x)$ then
find all the intervals
and integrate b/w them

$$\# I_1 = \int_0^1 (1-x^n)^k dx$$

$$I_2 = \int_0^1 (1-x^n)^{k+1} dx$$

$$\text{then } \frac{I_1}{I_2} = \frac{1+n(k+1)}{n(k+1)}$$

$$\# \int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_a^{2a} f(x) dx$$

$$\left\{ \begin{array}{l} \int_0^a f(x) dx = f(2a-x) \cdot f(x) \\ \int_a^{2a} f(x) dx = f(2a-x) \cdot f(x) \end{array} \right.$$

$$\# \int_a^b f(x) dx + \int_b^a f^{-1}(x) dx = bf(b) - af(a)$$

Limit as Sum

in question take $\left(\frac{1}{n}\right)$
common somehow

then write as
 $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \left(\frac{r}{n}\right) \rightarrow n^{\text{th}} \text{ term}$

take $\frac{r}{n} \rightarrow x$

$$\frac{1}{n} \rightarrow dx \quad \left| \lim_{n \rightarrow \infty} \sum_{r=1}^n a_r \right| \rightarrow \int$$

and then integrate

Lebrnitz theorem

$$I(x) = \int_{\phi(x)}^{\psi(x)} f(t) dt$$

$$= \left\{ I'(x) = f(\psi(x)) \psi'(x) - f(\phi(x)) \phi'(x) \right.$$

Indefinite Integration

$$\textcircled{1} \int \frac{1}{x} dx = \ln x + c$$

$$\textcircled{2} \int \frac{dx}{ax+b} = \frac{\ln(ax+b)}{a} + c$$

$$\textcircled{3} \int \sin x dx = -\cos x + c$$

$$\textcircled{4} \int \cos x dx = \sin x + c$$

$$\textcircled{5} \int \tan x dx = \ln(\sec x) + c$$

$$\textcircled{6} \int \cot x dx = \ln(\sin x) + c$$

$$\textcircled{7} \int \operatorname{cosec} x dx = \ln(\operatorname{cosec} x \cot x) + c$$

$$\textcircled{8} \int \sec x dx = \ln(\sec x + \tan x) + c$$

$$\textcircled{9} \int \sec^2 x dx = \tan x + c$$

$$\textcircled{10} \int \operatorname{cosec}^2 x dx = -\cot x + c$$

$$\textcircled{11} \int \sec x \tan x = \sec x + c$$

$$\textcircled{12} \int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$$

Substitution

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

SOME SPECIAL INTEGRAL

Date _____

Page No. _____

$$I = \int \sin^m x \cdot \cos^n x dx$$

Case ①

① m, n either even/odd
 $\Rightarrow$ which ever even power take as (t)

② m & n both odd
 $\Rightarrow$ take any (t)

③ m & n both even
 $\Rightarrow$ convert to trigonometric identities

Case ②

$$m+n-1 < 0$$

2. = put $\cot x$ or $\tan x$ as (t)

INTEGRATION BY PARTIAL FUNCTION

$$\text{TYPE - ①} = \int \frac{dx}{x(x+1)}$$

Write as $\frac{A}{x} + \frac{B}{(x+1)}$

- ② Find compare coeff
- ③ Integrate (Find A & B)

TYPE - ②

$$\int \frac{dx}{(x^2)(x+1)}$$

Write like this for power $\frac{A}{t} + \frac{B}{t^2} + \frac{C}{t+1}$ only when no repeating unit

Trick if its like $\frac{A}{x-1} + \frac{B}{(x+2)} + \frac{C}{(x+3)}$

then to find A Find x for denominator $x-1=0 \therefore x=1$ then put it in other denominators like $\frac{1}{(1+2)(1+3)} = \frac{1}{(3)(4)} = \frac{1}{12}$

Do for all

TYPE ③ $\rightarrow$ if denominator has quadratic

$$\frac{2x+7}{(x+1)(x^2+4)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+4)}$$

Note when linear $\int$ quadratic dx then $\rightarrow$

① Whenever $\ln(x)$ & x in same function and x is in denominator $[\ln(x) = b]$

Integration by parts

$$\textcircled{1} \int I II dx = I \int II dx - \left(\frac{d(I)}{dx} \int II dx \right) dx$$

$$\textcircled{2} \int e^x (f(x) + f'(x)) dx \Rightarrow e^x f(x) + C$$

IMPORTANT FORMULAS

$$\textcircled{1} \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$

$$\textcircled{2} \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$\textcircled{3} \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$\textcircled{1} \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$\textcircled{2} \int \frac{dx}{\sqrt{a^2+x^2}} = \ln |x + \sqrt{a^2+x^2}| + C$$

$$\textcircled{3} \int \frac{dx}{\sqrt{x^2-a^2}} = \ln |x + \sqrt{x^2-a^2}| + C$$

$$\textcircled{1} \int \sqrt{a^2-x^2} dx = \frac{1}{2} x \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$\textcircled{2} \int \sqrt{a^2+x^2} dx = \frac{x}{2} \sqrt{a^2+x^2} + \frac{a^2}{2} \ln |x + \sqrt{a^2+x^2}| + C$$

$$\textcircled{3} \int \sqrt{x^2-a^2} dx = \frac{x}{2} \sqrt{x^2-a^2} + \frac{a^2}{2} \ln |x + \sqrt{x^2-a^2}| + C$$

SUBSTITUTION

$$a^2+x^2 = x^2+x^2 = a \tan \theta \text{ or } a \sec \theta$$

$$a^2-x^2 = a \sin \theta \text{ or } a \cos \theta$$

$$x^2-a^2 = a \sec \theta \text{ or } a \csc \theta$$

Substitutions (contd)

$$\sqrt{\frac{a-x}{a+x}} \text{ or } \sqrt{\frac{a+x}{a-x}}$$

$$\therefore x = a \cos 2\theta$$

$$\sqrt{\frac{\alpha-x}{\beta-x}} \text{ or } \sqrt{(x-\alpha)(\beta-x)}$$

$$\therefore x = \alpha \cos^2 \theta + \beta \sin^2 \theta$$

When $\sin x$ & $\cos x$ are together ~~multiplied~~ the divide the whole functions numerator and denominator with $\cos x$'s highest power

$$\sqrt{a^2+x^2}$$

$$x = a \sin \theta$$

$$\sqrt{x^2-a^2}$$

$$x = a \sec \theta$$